جامعة بيرزيت

BIRZEIT UNIVERSITY

MATH DEPARTMENT

Test 2

MATH 1321

Summer Semester 2016

Student Name :____________

ID : ________

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(1)Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$

$\lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right|$

2 $= \lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{(x+2)^n} \right|$

2 $= \frac{|x+2|}{2} \lim_{n\to\infty} \frac{n}{n+1} = \frac{|x+2|}{2} < 1$

2 $-2 < x+2 < 2$

$-4 < x < 0$

$x = 0 \Rightarrow \sum \frac{(-1)^{n+1} 2^n}{n 2^n} = \sum \frac{(-1)^{n+1}}{n}$

2 Alternating harmonic series

converges by Lipinz test

2 $x = -4 \Rightarrow \sum \frac{(-1)^{n+1}(-2)^n}{n\, 2^n} = \sum \frac{(-1)^{2n+1}}{n} = -1 \sum \frac{1}{n}$

diverges by p test (harmonic series)

$\Rightarrow$ converges on $(-4, 0]$

(2)Find Taylor series expansion of $f(x) = \frac{1}{x}$, $a = 3$

and find the interval of convergence of the series and the sum of the series where it converges.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots$$

$$f(x) = \frac{1}{x} \qquad f(3) = \frac{1}{3}$$

$$f'(x) = \frac{-1}{x^2} \qquad f'(3) = \frac{-1}{9}$$

3

$$f''(x) = \frac{2}{x^3} \qquad f''(3) = \frac{2}{27}$$

$$f'''(x) = \frac{-6}{x^4} \qquad f'''(3) = \frac{-6}{81}$$

$$\frac{1}{x} = \frac{1}{3} - \frac{1}{9}(x-3) + \frac{2}{(27)(2)}(x-3)^2 - \frac{6}{(81)(6)}(x-3)^3 + \cdots$$

$$= \frac{1}{3} - \frac{1}{9}(x-3) + \frac{1}{3^3}(x-3)^2 - \frac{1}{3^4}(x-3)^3 + \cdots$$

3

$$= \frac{1}{3}\left[1 - \frac{x-3}{3} + \frac{(x-3)^2}{3^2} - \frac{(x-3)^3}{3^3} + \cdots\right]$$

$$= \frac{1}{3}\sum_{n=0}^{\infty}\frac{(-1)^n(x-3)^n}{3^n} = \frac{1}{3}\sum_{n=0}^{\infty}(-1)^n\left(\frac{x-3}{3}\right)^n$$

Absolutly,

Geometric Series converges if $|r| < 1$

$$\left|\frac{x-3}{3}\right| < 1 \Rightarrow |x-3| < 3$$

2

$$\Rightarrow -3 < x-3 < 3 \Rightarrow \boxed{0 < x < 6}$$

and it converges to $\frac{a}{1-r}$

2

$$= \frac{1/3}{1-\frac{x-3}{3}} = \frac{1/3}{\frac{3-(x+3)}{3}} = \frac{1}{x}$$

2

3) Estimate $\int_0^{0.1} e^{-x^2}\,dx$ with $|\text{error}| \leq 10^{-8}$

$$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

2

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \frac{x^{10}}{5!} + \cdots$$

$$\int_0^{0.1} e^{-x^2} = \int_0^{0.1} 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{24} - \frac{x^{10}}{120} - \cdots$$

2

$$= x - \frac{x^3}{3} + \frac{x^5}{5 \cdot 2!} - \frac{x^7}{7(3!)} + \frac{x^9}{9(24)} - \frac{x^{11}}{11(120)} \cdots \Big|_0^{0.1}$$

2

$$= 0.1 - \frac{(0.1)^3}{3} + \frac{(0.1)^5}{5 \cdot 2} - \frac{(0.1)^7}{7(6)} + \frac{(0.1)^9}{9(24)}$$

$$\text{since } \frac{(0.1)^7}{42} < 10^{-8}$$

$$\text{but } \frac{(0.1)^5}{10} = \frac{1}{10^6} \not< \frac{1}{10^8}$$

4

So

$$\int_0^{0.1} e^{-x^2}\,dx \approx 0.1 - \frac{(0.1)^3}{3} + \frac{(0.1)^5}{10}$$

$$\text{with } |\text{error}| \leq \frac{(0.1)^7}{42} < \frac{1}{(10^7)42} < \frac{1}{10^8}$$

4) (a) Replace the polar equation with equivalent Cartesian equation

$$r^2 sin2\theta = 1$$

2 $r^2 2\sin\theta\cos\theta = 1$

$r\sin\theta \; r\cos\theta = \frac{1}{2}$

2 $\boxed{xy = \frac{1}{2}}$

(b) Find the first three terms of the Binomial expansion of

$$\frac{1}{\sqrt[3]{1-\frac{x}{3}}}$$

2 $(1+x)^m = 1 + mx + \frac{m(m-1)}{2}x^2 + \cdots$

2 $\left(1+\left(\frac{-x}{3}\right)\right)^{-\frac{1}{3}} = 1 - \frac{1}{3}\left(\frac{-x}{3}\right) + \frac{-\frac{1}{3}\left(\frac{-4}{3}\right)}{2}\left(\frac{-x}{3}\right)^2 + \cdots$

2 $= 1 + \frac{x}{9} + \frac{2}{81}x^2 + \cdots$

5) Sketch the graph

$$r = \frac{1}{2} + sin\theta$$

symmetric about y-axis

θ	$r = \frac{1}{2} + \sin\theta$
0	$\frac{1}{2}$
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	1.21
$\frac{\pi}{3}$	1.37
$\frac{\pi}{2}$	1.5
$-\frac{\pi}{6}$	$\frac{1}{2} - \frac{1}{2} = 0$
$\frac{\pi}{4}$	$\frac{1}{2} - 0.71 \approx -0.21$
$\frac{\pi}{3}$	$0.5 - 0.87 \approx -0.37$
$\frac{\pi}{2}$	$\frac{1}{2} - 1 = -0.5$